Stability of infinite closed Jackson networks

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Consider the following particle system: a countable number of particles circulates in the network Λ of nodes being controlled by a routing matrix $P = \{p_{xy}\}_{x,y\in\Lambda}$: if there are some particles in the node $x \in \Lambda$ then one of them (particles do not have their individuality) instantly moves to a node $y \in \Lambda$ with probability p_{xy} at the moment of time distributed exponentially with parameter $\gamma(x)$, $\gamma = \sum\limits_{x\in\Lambda} \gamma(x) \leq \infty$. There is a finite number of particles in each node at the initial moment $t = 0$. The system informally described above is a generalisation of some particle systems introduced by Spitzer and then considered e.g. by Liggett et al. It can be also viewed as a closed Jackson-type network with infinitely many customers inside that connects this problem to the queueing theory. Under certain conditions on P and $\gamma(x)$ there exists the strongly continious Markov process $\{X(t)\}_{t\geq 0} = \{\, (X_y(t))_{y\in\Lambda}\, \}_{t\geq 0}$ on the state space $\left(\mathbb{Z}_+^\Lambda, \mathcal{B}_\Lambda\right)$ which describes the system: for any $y \in \Lambda$ $X_y(t)$ is a number of particles in y at time t ($\mathbb{Z}_+$ being the set of all $n \geq 0$, $n \in \mathbb{Z}$). One can say that the system is non-ergodic if there does not exist such unique stationary measure μ that for any distribution of $X(0)$ the measures correspondent to $X(t)$ weakly converge to μ as $t \to \infty$. In this report the long-time behaviour of the system is studied: cases of non-ergodicity are explored, the class of stationary measures is described. Some further developments are made in the case $\gamma < \infty$: then one can consider an embedded Markov chain that facilitates the proofs. Here the results about the stochastical boundedness of $X_y(t)$ for a concrete $y \in \Lambda$ and the existence of a functional of $X(t)$ that appears to be a supermartingale are obtained.

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